

# Çankaya University – ECE Department – ECE 376

Student Name :  
Student Number :

Duration : 2 hours  
Open book exam

## Questions

1. (35 Points) A) A message signal of  $v(t) = a_0 g_T(t) + a_1 g_T(t - T_b)$ , where  $a_0, a_1 = \mp 1$ ,  $T_b = 0.5$  msec. and  $g_T(t) = \begin{cases} 1 & \text{for } 0 \leq t \leq T_b \\ 0 & \text{otherwise} \end{cases}$ , gets multiplied by a PN sequence of

$$c(t) = \sum_{n=0}^N c_n p(t - nT_c), \text{ where } c_n = \mp 1, T_c = 0.5 \mu\text{sec. and } p(t) = \begin{cases} 1 & \text{for } 0 \leq t \leq T_c \\ 0 & \text{otherwise} \end{cases}$$

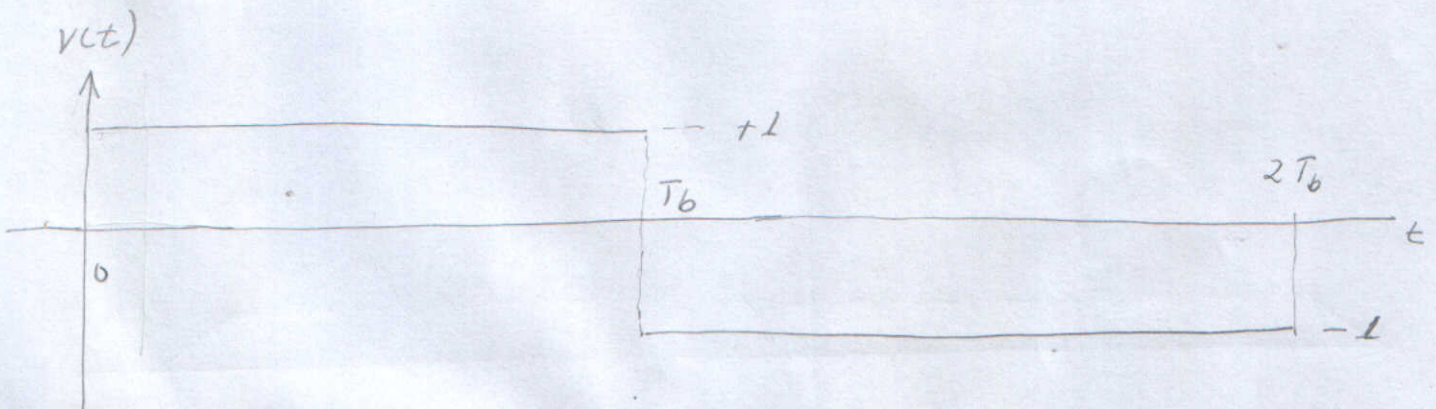
Considering one  $T_b$  duration covers the entire time duration of  $c(t)$ , what should be  $N$ ? For the given case, plot time waveforms and frequency spectrums, appropriately marking the numeric values on the time and propagation axes.

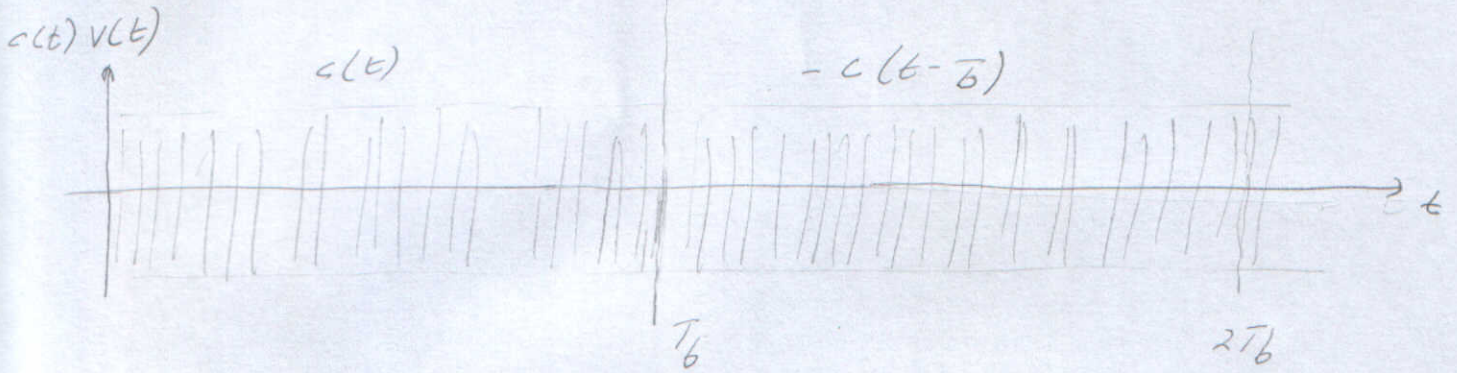
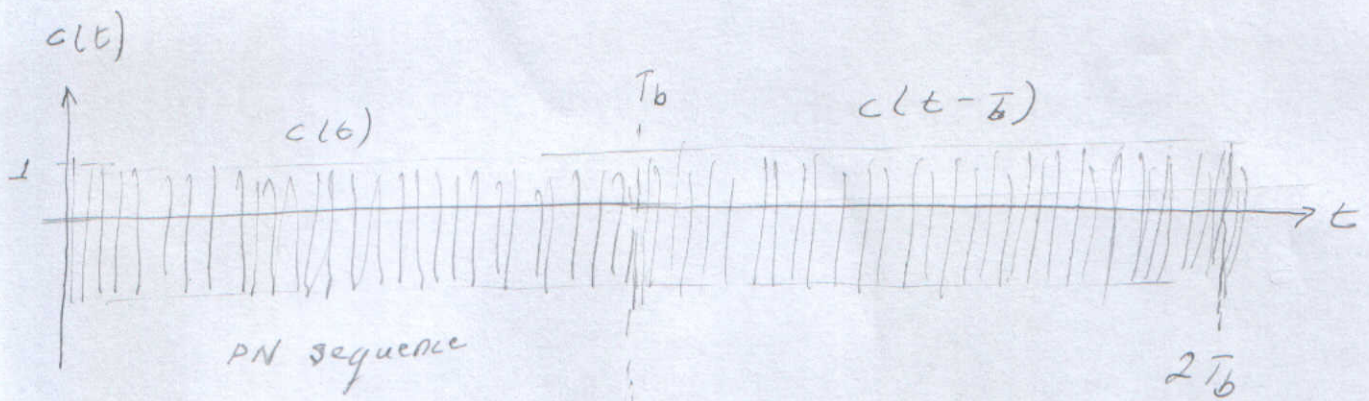
B) For a DS spread spectrum system employing five users (i.e., five different message signals), draw the transmitter and receiver block diagrams explaining the function of each block.

Solution: A) Since one  $T_b$  duration should cover one  $c(t)$  then  $N = \frac{T_b}{T_c} = L_c = 1000$

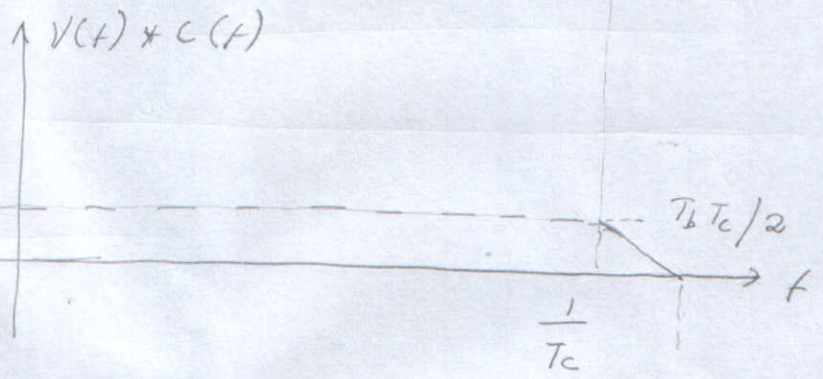
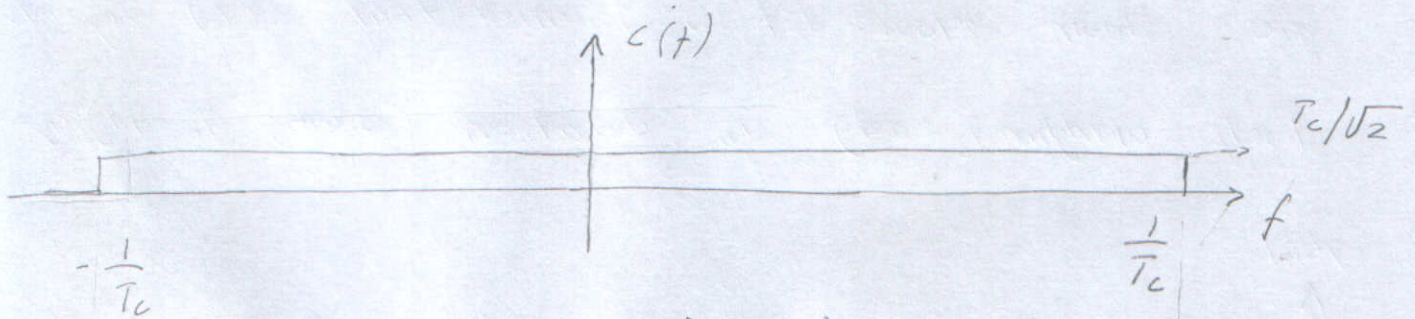
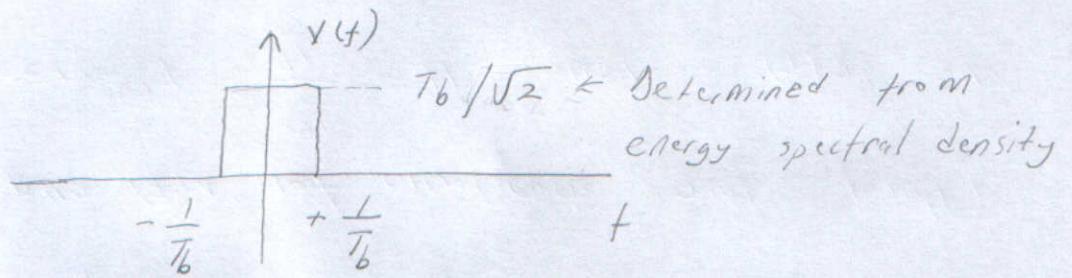
Typical time waveforms for  $v(t)$  and  $v(t)c(t)$

are shown below. It is assumed that  $c(t)$  repeats itself within every  $T_b$  duration (set,  $a_0 = +1, a_1 = -1$ )





Approximate spectrums (rounded to window functions)



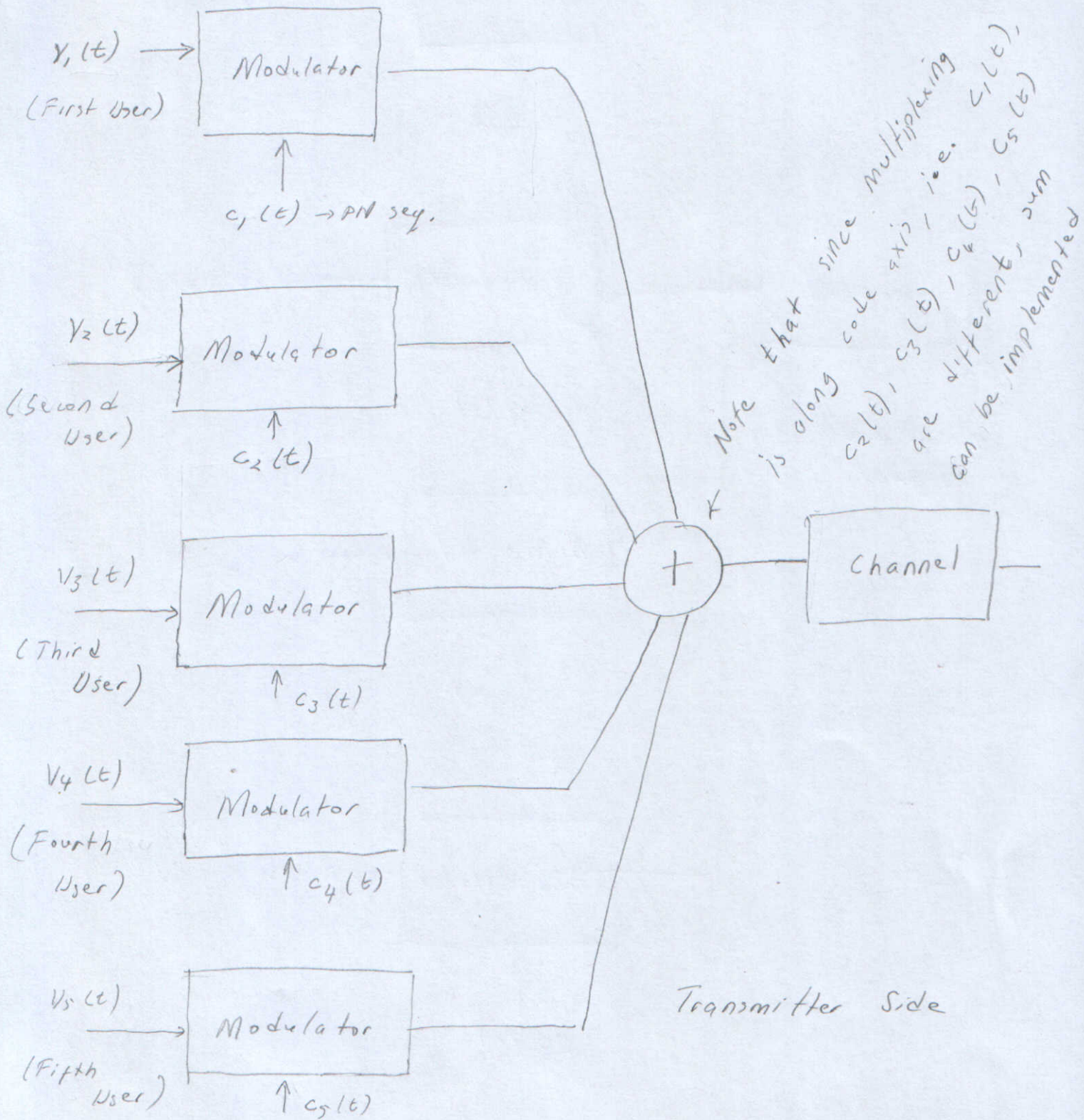
$$-\frac{1}{T_c} - \frac{2}{T_b}$$

$$\frac{1}{T_c} + \frac{2}{T_b}$$

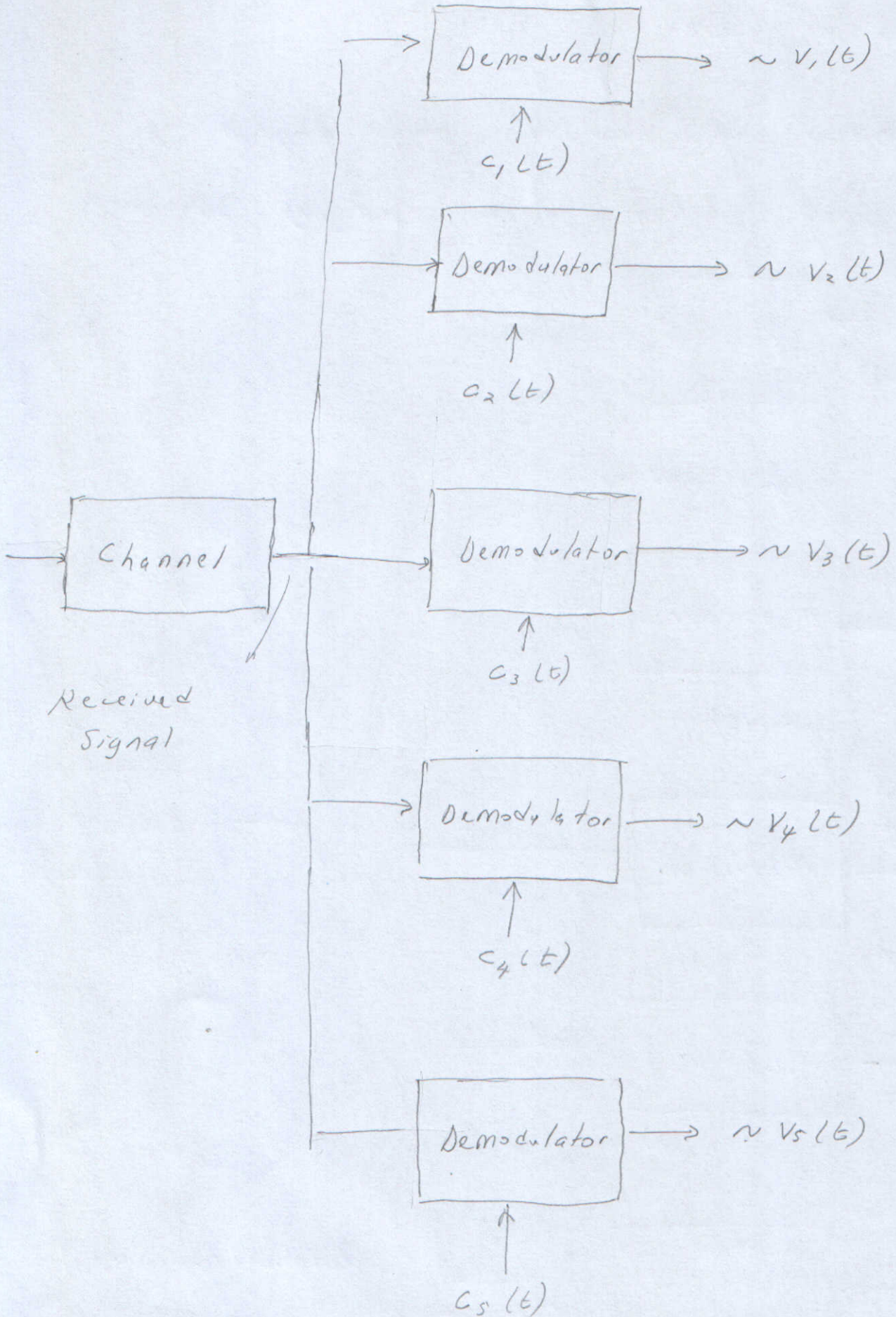
Addendum p. 2

B) Below we show the block diagram of

DS spread spectrum system employing five users



Receiver Side



2. (35 Points) An OFDM system with four sub-carriers is to be designed for a 4 QAM signal with a rate of 1 k symbols / sec. By choosing the appropriate sub-carriers and the frequency spacing between them, show that each symbol of 4 QAM, when placed on a sub-carrier, is orthogonal to all other three symbols, if integrated over a symbol period, even if the symbols have different phase shifts. Draw and explain the functioning of block diagram of transmitter and receiver for this OFDM system.

Solution : Since symbol rate is 1 k symbols/sec

$T_s = 10^{-3}$  sec = 1 m sec and a symbol is placed

on the same carrier every  $4T_s$ , then  $T = 4T_s = 4$  m sec.

So (excluding a shift of  $f_c$ ), if  $f_1 = 1$  kHz  $\rightarrow$  1st sub carrier

$$f_k - f_1 = \frac{n}{T_s} \quad \text{where } k = 2, 3, 4, \quad n = 1, 2, 3$$

sub carriers

Then we find  $f_2 = 2$  kHz,  $f_3 = 3$  kHz,  $f_4 = 4$  kHz

Note that the spacing between subcarrier can also

be adjusted with  $T$ , in this case by keeping  $f_1 = 2$  kHz

$$f_2 = 1250 \text{ Hz}, \quad f_3 = 1500 \text{ Hz}, \quad f_4 = 1750 \text{ Hz}$$

Now perform the integration below

$$\int_0^{T_s} \cos(2\pi f_k t + \phi_k) \cos(2\pi f_j t + \phi_j) dt$$

$$= \frac{1}{4\pi(f_k - f_j)} \sin[2\pi(f_k - f_j)t + \phi_k - \phi_j] \Big|_0^{T_s} +$$

$$\frac{1}{4\pi(t_k + t_j)} \sin [2\pi(t_k + t_j)t + \phi_k + \phi_j] \Big|_0^{T_s}$$

$$= \frac{1}{4\pi(t_k - t_j)} \left\{ \sin [2\pi(t_k - t_j)T_s + \phi_k - \phi_j] \right.$$

$$- \sin(\phi_k - \phi_j) \left. \right\} + \frac{1}{4\pi(t_k + t_j)} \left\{ \sin [2\pi(t_k + t_j)T_s + \phi_k + \phi_j] \right.$$

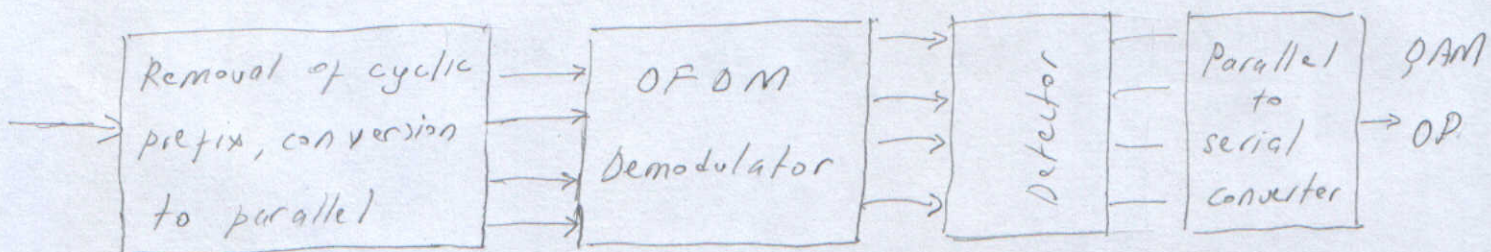
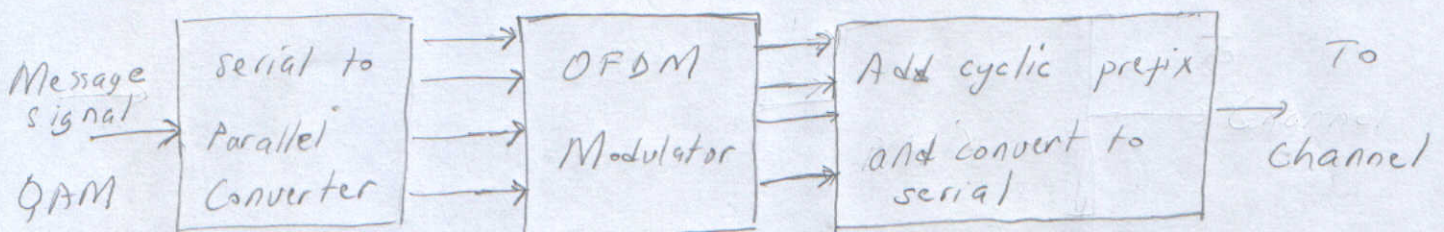
$$- \sin(\phi_k + \phi_j) \left. \right\}$$

where the first difference of sines reduces to

zero since  $t_k - t_j = \frac{n}{T_s}$ , but for second group of sines to evaluate to zero it must be that

$$t_j = \frac{n}{T_s} \text{ or the starting } t_0 = \frac{n}{T_s}.$$

Block diagram of OFDM system is shown below



3. (30 Points) Answer the following questions as **True** or **False**. For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer.

a) Prob. of error in 4 QAM is higher than prob. of error in 8 PSK: *If compared on equal power (or equal energy) basis, this cannot be true, since the constellation of 8 PSK is more crowded than 4 QAM.*

b) In MF detection, distance metrics,  $D(\mathbf{r}, \mathbf{s}_m)$  calculation gives the correlation of received signal vector  $\mathbf{r}$  with the axis of basis functions: *False, since  $D(\mathbf{r}, \mathbf{s}_m)$  gives the distance between the signal vector  $\mathbf{r}$  and all signal vectors of the constellation*

c) The number of dimensions in a set of signals can be determined by eye inspection:

*Partially true, but if this is not possible, then Gram-Schmidt orthogonalization procedure must formally be applied*

d) FSK is a combination of QAM and PSK: *False, QAM can be considered to be a combination of ASK and PSK.*

e) For detection, receiver attempts to determine the constellation diagram sent by transmitter:

*False, the receiver already knows the constellation diagram, the receiver attempts to determine which of the signals in this constellation diagram was transmitted*

f) After successful MF detection, we can say that we have completely eliminated noise from the received signal:

*True, if the detection is successful, then MF detector outputs a clean copy (without noise) of transmitted signal.*